

Exploratory worksheet

January 29th

For each of the statements below, determine whether they are true or false: give a proof if they are true or a counterexample if false. (*Warning: Some statements might be difficult to prove/disprove, in which case you should guess whether you think the result is true or false and provide some computational evidence.*)

1. Every prime number is of the form $6k + 1$ or $6k - 1$ for some integer k .
2. Let $n > 2$ be an integer.
 - (a) If $n \mid 2^{n-1} - 1$ then n is prime.
 - (b) If n is prime, then $n \mid 2^{n-1} - 1$.
3.
 - (a) There are infinitely many primes of the form $3k + 1$.
 - (b) There are infinitely many primes of the form $3k + 2$.
4. There are more primes of the form $3k + 2$ than of the form $3k + 1$. More specifically, if

$$\pi_1(n) = \#\{\text{primes } p \leq n \text{ of the form } 3k + 1\},$$

$$\pi_2(n) = \#\{\text{primes } p \leq n \text{ of the form } 3k + 2\},$$

then $\pi_1(n) \leq \pi_2(n)$ for any integer $n \geq 2$.